An Expository Model of Credit Rationing*

This paper presents a simple exposition of the main results behind more complicated credit rationing models. It uses a standard, static model of the banking firm to analyze the effects of changing the interest rate on loans and the probability of repayment. Using simple elasticity arguments we demonstrate how “natural” credit rationing can arise.

Based on the pathbreaking work of Stiglitz and Weiss (1981), a considerable amount of progress has been made in explaining why credit rationing can arise “naturally” in competitive markets. Natural credit rationing occurs when borrowers are not permitted to borrow as much as they would like at the current market rate of interest or when, among a group of borrowers who look the same, some are able to obtain credit while others are not. This arises because the quantity of loans extended by profit maximizing firms is not continually increasing in the loan rate, which implies that increasing the market rate of interest may not eliminate the excess loan demand.

While the models in this area have been around for quite awhile, it is still reasonably difficult for the general reader or first-year graduate student to derive the fundamental results in a simple and intuitive fashion. Furthermore, no simple models exist to demonstrate the insights of the credit rationing literature to undergraduate economics majors. One reason for this is the models do not lend themselves well to standard supply theory due to the complexity of the problem facing the banking firm. We resolve this problem by developing a simple model of the banking firm which incorporates the key insights of the credit rationing literature. It is especially useful since it requires a level of analysis and basic concepts, such as elasticity, that anyone with a good understanding of intermediate microeconomic theory can understand.

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1By natural we mean it does not arise because of artificially imposed credit market constraints such as interest rate ceilings.
1. A Model of the Banking Firm

Consider a perfectly competitive market for loans and deposits. The representative banking firm chooses loan volume (either the number of loans or loan size) to maximize profits taking the market rates of interest on loans and deposits as given. A crucial feature of credit-rationing models is that loans may not be repaid due to default. Therefore, let $p$ denote the probability of loan repayment. In order to derive a standard marginal cost curve, it is assumed that banks incur variable costs of operating and for simplicity we assume the bank's operating costs can be specified as a function of their loan volume. This assumption is not crucial for the typically credit-rationing result; it merely aids in using standard theory of the firm analysis. Finally, banking firms are assumed to be risk neutral.

Given the assumptions above, the firm chooses loans, $L$, to

$$
\max_{L} II = pLr_L - Dr_D - (q/2)L^2
$$

s.t. 

$$
D = bL, \quad b \geq 1
$$

where $r_L = 1 + R_L$ and $r_D = 1 + R_D$ are respectively the gross interest rates on loans and deposits, $L$ is loan volume, $D$ represents total deposits, $q$ is a cost parameter of servicing loans, and $b$ is the deposit to loan ratio and is equal to one over one minus the reserve requirement. The first term in (1) is expected total revenues while the latter terms represent total costs. Substituting the constraint into (1) and maximizing with respect to $L$ yields the firm's loan supply curve

$$
L^s = (pr_L - br_D)/q.
$$

Loan supply is positive if the numerator is positive which requires the expected marginal revenue per loan to exceed the firm's "after reserve requirement tax" interest payment per deposit.

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3This is a common assumption in models of the banking firm. For a textbook treatment, see Miller and VanHoose (1992).

4Loans are assumed to be simple one-period debt contracts; that is, they are bonds, not equities or some other type of state-contingent contract. However, this presumes that debt contracts are the optimal form of lending. Starting with the work of Townsend (1979), a significant literature has developed explaining why debt contracts are superior to equity contracts, particularly for the types of loans that banks typically make. For simplicity, we assume debt contracts are the optimal form of repayment. For a model that combines the optimal debt contract literature with the credit-rationing literature, see Williamson (1987).

5Note that if $q = 0$, then $L^s$ is either zero or infinity. This reflects the existence of an arbitrage opportunity from either the lending or depositing side and one can make an infinite profit by either lending or depositing depending on the sign of the numerator. A zero-profit equilibrium
Differentiating (2) with respect to \( r_L \) gives the inverse slope of the loan supply curve. Inverting this derivative yields the slope of a standard supply curve

\[
\frac{\partial r_L}{\partial L^s} = q/p > 0.
\]

The quantity of loans supplied is unambiguously increasing in the loan rate even when the possibility of default exists. A decrease in the exogenous probability of repayment steepens the slope of the loan supply curve and decreases the quantity of loans supplied at a given loan rate but does not change the sign of the supply curve slope. Consequently, for any downward sloping loan demand curve, an interest rate will exist such that no excess demand for loans exists, hence "natural" credit rationing will not occur.

2. Credit Rationing

The key feature of most credit-rationing models is that the probability of loan repayment is a function of the loan rate, whereby increases in the loan rate cause a decrease in the probability of repayment. Two basic arguments are typically given for this relationship. First, in the case of symmetric ex ante information, where both the lender and the borrower have the same information regarding the expected return on a borrower’s investment project, increasing the interest rate increases the interest cost of borrowing and makes it more difficult to repay both principle and interest out of a given return on the investment project. This line of reasoning is the basis for credit rationing models of Hodgeman (1960) and its descendants.

A second argument for this relationship between the loan rate and the probability of loan repayment is based on information asymmetries between the borrower and lender. This is the key to the genre of models following Stiglitz and Weiss (1981). The basic idea is that there are "safe" borrowers, whose investment projects have low risk but low rates of returns, and "risky" borrowers, whose projects have high risk but high rates of return. However, only the borrower knows whether they have a safe or risky project; banks do not. In this case, an increase in the loan rate gradually induces borrowers with "safe" projects to drop out of the market since safe projects produce a lower rate of return but are less risky. This leaves only the borrowers with risky projects who default more frequently. Although the banking firm cannot determine which borrowers have safe and risky projects, it does know that

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would then require the expected return on loans to equal the deposit rate. This is the way most credit-rationing models are constructed.
an increase in the loan rate will cause safe borrowers to drop out faster than risky borrowers and is thus aware that the aggregate repayment probability decreases as the loan rate increases.

The situation above is an adverse selection problem. The problem above also arises if a moral hazard problem exists whereby borrowers get to select between safe and risky projects once they receive the loan. As the loan rate increases, borrowers will prefer to switch from safe projects to riskier projects in order to pay the higher interest rate on the loan but lower the repayment probability in the process. Once again, banks are aware of this possibility and realize the repayment probability will decrease as the loan rate increases.

The key point from this discussion is that the repayment probability is not exogenous, as assumed above, but rather is dependent on the loan rate. To capture this idea let \( p = f(r_L) \) with \( \frac{\partial p}{\partial r_L} = f'(r_L) < 0 \). In the results above, Equation (2) is unchanged since the competitive banking firm chooses \( L \) not \( r_L \) to maximize profits. Hence, Equation (2) still gives us the firm's loan supply curve. The change occurs when we find the slope of the loan supply curve. Differentiating (2) with respect to \( r_L \), keeping in mind the relationship between \( p \) and \( r_L \), rearranging terms and inverting yields

\[
\frac{\partial r_L}{\partial L} = \frac{q/p}{1 + \left( \frac{\partial p}{\partial r_L} \right) \left( \frac{r_L}{p} \right)}
\]

\[
= \frac{q/p}{1 + e_p}^{-1},
\]

where \( e_p \) is the loan rate elasticity of the repayment probability.\(^5\) This supply curve slope has an ambiguous sign since \( e_p \) is negative by assumption. The intuition behind this ambiguity provides the key to understanding the credit-ratieving result. An increase in the loan rate has two effects on the expected revenues of the bank. First, a 1% increase in the gross loan rate, holding the repayment probability constant, causes a 1% increase in expected revenues, which explains the 1 in the brackets. This effect induces the firm to increase the quantity of lending. However, a 1% increase in the gross loan rate causes an \( e_p \) percent decrease in the probability of repayment and an \( e_p \) percent decrease in expected revenues, which makes the firm want to decrease the quantity of lending. The actual change in the quantity of lending depends on which effect is stronger.

It is reasonable to believe that, starting at a relatively low loan rate, an increase in the loan rate will not cause a great deal of financial burden on borrowers or cause a large number of safe borrowers to drop out of the market. Hence, at low loan rates the first effect dominates and the loan supply

\(^5\)The appropriate expression in terms of the net loan rate \( R_L \) would be

\[
\frac{\partial R_L}{\partial L} = \frac{q/p}{1 + \left( \frac{\partial p}{\partial R_L} \right) \left( \frac{R_L}{p} \right) \left( 1 + R_L/R_L \right)}
\]
curve is upward sloping. But as the loan rate continues to increase, it reaches a point where the marginal gain in expected revenues from having a higher loan rate is just offset by the marginal loss in expected revenues caused by a higher expected number of defaults. At this point, the two effects cancel each other out and the loan supply curve becomes vertical. At relatively high loan rates the second effect comes to dominate as the interest payments become crushing and seriously impair borrowers’ ability to repay. This continues until, at some loan rate, the probability of repayment is so low that the numerator in Equation (2) becomes negative and the bank stops extending loans. In this range of high loan rates, the slope of the loan supply curve is negative as banking firms actually contract credit in response to higher loan rates. Combining all three of these ranges produces a backward-bending loan supply curve.

The situation described above will arise, for example, if the probability of repayment is given by

\[ p = \alpha - \beta r_L^2, \quad 0 \leq r_L \leq (\alpha/\beta)^{1/2}, \quad 0 < \beta < \alpha \leq 1. \]  

(5)

In this case, \( e_p \) is

\[ e_p = -2\beta r_L^2/\left(\alpha - \beta r_L^2\right). \]  

(6)

which is equal to \(-1\) at the point \( r_L = (\alpha/3\beta)^{1/2} \). Substituting (5) into (2) and maximizing with respect to \( r_L \) shows that loan volume is maximized at this same value of \( r_L \).

This ambiguous relationship between the loan rate and the quantity of loans supplied captures the essence of credit-rationing models. Aggregating over the individual loan supply curves gives us the market loan supply curve. Examining Figure 1 we now can see how credit rationing arises. For any downward-sloping loan demand, such as loan demand 1, intersecting the backward-bending loan supply curve in the range \( L > 0 \), an equilibrium interest rate exists and natural credit-rationing will not occur. However, if the loan demand curve were to increase to loan demand 2, then there will not be an interest rate which eliminates the excess loan demand and credit rationing will arise. Profit-maximization suggests the banking firm will extend loans only out to the point where the loan supply curve bends back since raising loan rates above this point actually reduces profits.\(^6\) Therefore, in the absence of a policy which shifts the loan supply curve to the right, or in the

\(^6\)This can be verified from our example by substituting (2) and (5) into (1) to obtain \( II(r_L) \). This expression will be maximized when \( r_L = (\alpha/3\beta)^{1/2} \), which is the value of \( r_L \) at which the loan supply curve bends back.
absence of a decrease in loan demand, a disequilibrium, not caused by artificial constraints in the market, will arise and persist.  

3. Conclusions

The general ideas behind credit-rationing are easy to verbalize but not easy to demonstrate. We have provided a simple model of credit-rationing to fill this void and hopefully have created a pedagogical framework to help convey the logic behind “natural” credit-rationing to a wider audience of economists and students.

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7An example of a policy shift would be lowering the reserve requirement on deposits. This would decrease the value of b and, from (2), would increase the quantity of lending at any loan rate.
References