Room-temperature steady-state optomechanics entanglement on a chip

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A potential experimental system, based on the Silicon Nitride (SiN) material, is proposed to generate steady-state room-temperature optomechanical entanglement. In the proposed structure, the nanostring interacts dispersively and reactively with the microdisk cavity via the evanescent field. We study the role of both dispersive and reactive coupling in generating optomechanical entanglement, and show that the room-temperature entanglement can be effectively obtained through the dispersive couplings within the reasonable experimental parameters. In particular, we find, in the high Temperature (T) and high mechanical qualify factor (Q_m) limit, the logarithmic entanglement depends only on the ratio T/Q_m. This means that improvements in the material quantity and structure design may lead to more efficient generation of stationary high-temperature entanglement.

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Introduction.- The quantum entanglement not only represents one of the most interesting fundamental phenomena in quantum mechanics, but also is regarded as very important resource in quantum computation and quantum information processing [1]. It has been a focus in quantum mechanics since the seminal work of Einstein-Podolsky-Rosen (EPR) gedanken experiment [2]. Nowadays, quantum entanglement has been observed in various quantum systems, such as photons, atoms, and quantum dots [3]. Although it is believed that large decoherence effect would mask quantum signatures of macroscopic objects, great efforts are still dedicated to macroscopic quantum entanglement, in order to exploring the boundary between quantum and classical mechanics.

Recently, rapid progress in nanofabrication technologies offers novel opportunities to study macroscopic quantum entanglement [4]. Especially, combined with mature optical technologies, the emerging optomechanics enable precisely controlling the motion of mechanical oscillators through optical radiation force or gradient force [5]. It was proposed that the mechanical vibration could be entangled with the optical field in various optomechanics systems, such as Fabry-Pérot cavity with a movable mirror [6] or nanomembranes [7]. It could also be extended to microwave cavities [8]. Unfortunately, quantum entanglement in these systems is very sensitive to the temperature, typically only valid in cryostat. Up to now quantum entanglement related to mechanical modes has not been observed in any experiment. One of the key question arises: Is it possible to entangle a light beam and macroscopic objects at room temperature?

In this paper, we propose a potential microcavity-nanostring system to generate the room-temperature optomechanical entanglement. The system is based on the Silicon Nitride material, the nanostring interacts dispersively and reactively with the microdisk cavity via the evanescent field. It is shown that entanglement can be effectively generated through dispersive couplings within the current experimental parameters, and can be preserved at room temperature or even at a high temperature. We analyzes the the dependence of entanglement on the temperature and mechanical dissipation, and find that the logarithmic entanglement is function of the ratio T/Q_m in the high-Q_m and high-temperature limit T. This demonstrates that the improvements in the material quantity and structure design may lead to more efficient generation of stationary room-temperature entanglement.

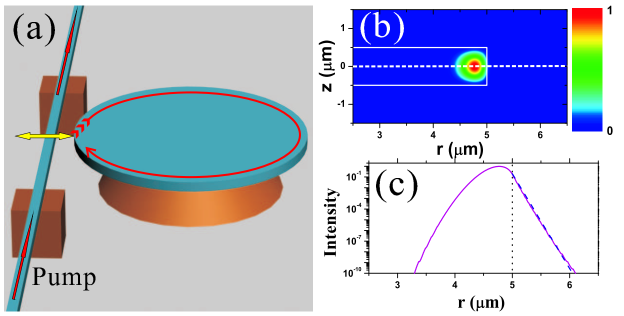

FIG. 1: (color online) (a) Schematic illustration of a nanostring coupled to a high-Q WG microcavity. The microcavity is evanescent-field driven by the waveguide, which is also act as a high-Q mechanical oscillator. (b) The normalized field $|E|^2$ distribution of cross section of the SiN microdisk with diameter $\Phi = 10\mu m$ and thickness $t = 1\mu m$ (c) The normalized field $|E|^2$ distribution at $z = 0$ (dashed line in (b)), the evanscent field is fit to a exponential decay curve with decay length $l_0 = 100nm$.

System.- As shown in Fig.1(a), the system consists of a nanostring oscillator and a microdisk cavity, fabricated by SiN, which can be integrated on single chip. This structure have been experimentally realized in silicon chip, but the performance is limited by low mechanical quality factor (Q_m) and low optical quality factors (Q_o) [9]. Similar structure with a nanostring closed to a silica toroid has also been studied by Anetsberger et al.[10], which shows displacement sensitivity beyond the standard quantum limit. However, it is a challenge to precisely control the gap between the nanostring and cavity in experiments[10]. Compared with these studies[9, 10], the experimental system proposed in this paper has the following advantages: (1) The fabrication technology of SiN device is compatible with silicon, and permits further expendability of on-chip optical components and opto-electronics elements. (2) The SiN is transparent for a wide band, and has relative high refractive index. Thus dielectric microcavity based on SiN can realize the great confinement of light, possess ultrahigh qualify factor and small mode volumes. Currently, the whispering gallery (WG) modes with qualify factor Q_o up to about 4×10^6 have been demonstrated experimentally[11]. (3) The mechanical properties of SiN is excellent, so that we can fabricate strained nanostrings with mechanical qualify factor $Q_m > 10^6$ at room temperature[12]. The large Q_o significantly enhanced the light-matter interaction at low input intensity, and large Q_m permits very long coherence time. Therefore, our system provides a unique combination of ultrahigh mechanical and optical quality factor, and holds great potential for quantum optomechanics.

To study the interaction between the microcavity and nanostring, we plot the mode field distribution of microdisk in Fig.1(b), which with diameter $\Phi = 10\mu m$ and thickness $t = 1\mu m$. It is shown that the WG modes is well confined in the microdisk, with radiation related $Q > 10^8$. A small portion of energy outside the cavity form the evanescent waves, which is exponentially decay with the distance from the dielectric interface (x), and can be approximately describe by $E_c(x) = E_c(0)e^{-x/l_0}$, where l_0 is the decay length. In Fig.1(c), we plot the normalized field $|E|^2$ distribution at $z = 0$, and find that the evanscent field is fit to a exponential decay curve with decay length $l_0 = 100nm$. In addition, as shown in Fig.1(a), the nanostring, as a waveguide, transfers light into and out from the microcavity. When the nanostring is placed in the evanescent-field of the WG mode, the mechanical oscillator will be attracted to the cavity through the gradient force, and the displacement of the nanostring also gives a backaction on the WG mode, and changes both the resonance frequency (dispersive coupling, DC) and energy decay rate (reactive coupling, RC).

From the coupling mode theory [13], the frequency shift $\delta\omega$ to the cavity resonance and the coupling strength κ_1 between the cavity mode and waveguiding mode can be expressed as $\delta\omega(x) \approx \delta\omega(0)e^{-2x/l_0}$, and $\kappa_1(x) \approx \kappa_1(0)e^{-2x/l_0}$. Thus, for a small displacement x around steady position, $\omega_c(x) \approx \omega_c(0) - dx$, and $\kappa_1(x) \approx \kappa_1(0) + rx$,where d and r describe the dispersive and reactive coupling strengths between the mechanical and optical modes, respectively, and both of them could be controlled experimentally by adjusting the size of disk and nanostring and gap between them.

Model.- The Hamiltonian of the coupled microcavity and nanostring system is given by [14, 15]

$$H = \hbar\omega_c a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2 + q^2) - \hbar D a^\dagger a q \quad (1)$$
$$+ i\hbar(\sqrt{2\kappa_1(x)} + Rq/\sqrt{2\kappa_1(x)})E(e^{-i\omega_l t}a^\dagger - e^{i\omega_l t}a),$$

where Bose operators a and $a^\dagger$ represent the annihilation and creation operators of the cavity mode with frequency ω_c and intrinsic loss κ_0. For the nanostring, we define dimensionless position and momentum operators $q = \sqrt{m\omega_m/\hbar}x$, $p = \sqrt{m/\hbar\omega_m}\dot{x}$, where m stands for effective mass of the nanostring with a resonant frequency ω_m. The corresponding dispersive and reactive coupling strength are normalized by zero point fluctuation $\sqrt{\hbar/m\omega_m}$: $D = d/\sqrt{m\omega_m/\hbar}$ and $R = r/\sqrt{m\omega_m/\hbar}$. The system is driven by a coherent laser with frequency ω_l and power P, corresponding to the driving field $E = \sqrt{P/\hbar\omega_l}$.

By including the noise on both the mechanical ($\xi(t)$) and optical ($a_{in}(t)$) modes, we obtain the quantum Langevin equations (QLE) in the reference frame rotating with frequency ω_l,

$$\dot{q} = \omega_m p, \quad (2a)$$
$$\dot{p} = -\omega_m q - \gamma_m p + G a^\dagger a - iR/\sqrt{2\kappa_1}E(a^\dagger - a)$$
$$- iR/\sqrt{2\kappa}(a^\dagger a_{in} - a a_{in}^\dagger) + \xi(t), \quad (2b)$$
$$\dot{a} = -i\Delta a - (\kappa + Rq)a + iGqa$$
$$+ (\sqrt{2\kappa_1} + Rq/\sqrt{2\kappa_1})E + \sqrt{2\kappa}a_{in}(t), \quad (2c)$$

where $\kappa = \kappa_0 + \kappa_1$, $\Delta = \omega_c - \omega_l$, and the noise correlation function are $\left\langle a_{in}(t)a_{in}^\dagger(t')\right\rangle = \delta(t-t')$, and $\langle\xi(t)\xi(t')\rangle = \gamma_m(2\bar{n}+1)\delta(t-t')$. Here $\bar{n} = 1/(\exp(\hbar\omega_m/k_B T) - 1)$ is the mean thermal phonon number, where k_B and T denote Boltzmann constant and temperature. By linearizing operators around the steady state values ($o = o_s + \delta o$, o is a system operator), we obtain the QLE for the fluctuation operators

$$\dot{f}(t) = Mf(t) + n(t), \quad (3)$$

where $f = (q, p, X, Y)^T$ is the vector of fluctuation operators, n is corresponding noises and the matrix

$$M = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & -\gamma_m & DX_s & -R\frac{E}{\sqrt{\kappa_1}} \\ R\frac{E}{\sqrt{\kappa_1}} - RX_s & 0 & -\kappa_s & \Delta_s \\ DX_s & 0 & -\Delta_s & -\kappa_s \end{pmatrix}. \quad (4)$$

Here we introduce the cavity field quadratures $X = (a + a^\dagger)/\sqrt{2}$ and $Y = i(a^\dagger - a)/\sqrt{2}$. By carefully choosing the phase of driving light, we set $\mathrm{Im}a_s = 0$, then $X_s = \sqrt{2}a_s$ and $Y_s = 0$. And $\Delta_s = \Delta - Dq_s$, $\kappa_s = \kappa + Rq_s$ the detuning and decay rate of cavity mode at steady state.

When stability condition is fulfilled, which can be derived by employing the Routh-Hurwitz criterion [16],

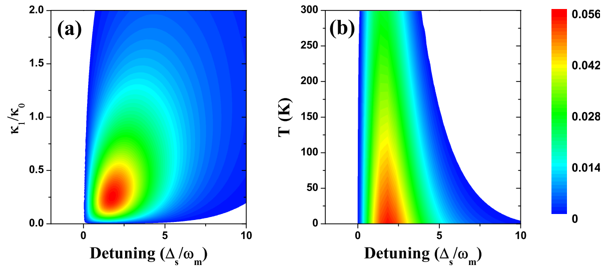

FIG. 2: (color online) Logarithmic negativity E_N against coupling parameter κ_1/κ_0, temperature T and detuning Δ_s, with $T = 0.05K$ (a), $\kappa_1/\kappa_0 = 0.3$ (b). The blank region means $E_N = 0$, the input power is fixed at 100mW.

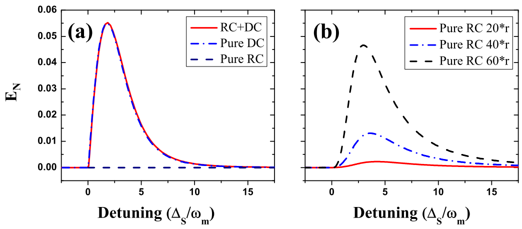

FIG. 3: (color online) E_N vs. detuning Δ_s for (a) realism condition contain both reactive coupling (RC) and dispersive coupling (DC), compares to the pure RC with $g = 0$ and DC with $r = 0$, (b) pure RC with $g = 0$, enhanced reactive coupling strength by 20, 40 and 60 times. The input power $P = 100$ mW, temperature $T = 0.05$ K and $\kappa_1/\kappa_0 = 0.3$.

we can solve the stochastic differential equation for the steady-state correlation matrix (V)

$$M \cdot V + V \cdot M^T = -I, \tag{5}$$

where V is defined as $V_{ij} = \langle f_i f_j + f_j f_i \rangle /2$, and $I = Diag\left[0, \gamma_m(2\bar{n}+1) + (R/\sqrt{2\kappa_1})^2 X_s/2, \kappa, \kappa\right]$. We represent V in the 2×2 block form,

$$V = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}, \tag{6}$$

then the entanglement of between optical field and mechanics oscillation can be quantified by the logarithmic negativity E_N [17]:

$$E_N = Max[0, -\frac{1}{2}\ln 2(\Sigma - \sqrt{\Sigma^2 - 4\det V})], \tag{7}$$

where $\Sigma = \det A + \det B - 2\det C$.

Results and Discussion.- To model the quantum entanglement between light field and motion of nanostring, we adapt the parameters closed to recent experiments. The microdisk with $Q_o = 4 \times 10^6$ is working at wavelength $\lambda = 850$ nm, which have been demonstrated experimentally[11]. The nanostring is chosen with $\omega_m = 15$ MHz, $m = 2$ pg, and $Q_m = 10^6$ at room temperature, which is achievable by current technology [12]. We set the coupling strength $r = 2\kappa_1/l_0$ with $l_0 = 100$ nm, and $g = 50$ MHz/nm. All calculations of E_N are performed under the stability condition.

In Fig. 2(a), we calculated the dependence of E_N on the cavity mode detuning Δ_s and the coupling condition κ_1/κ_0 at low temperature ($T = 0.05$ K). It can be seen that entanglement appears at blue detuning, and there exist a optimal coupling strength $\kappa_1/\kappa_0 \approx 0.3$ for entanglement. This can be understood as follows: From the Eq.(4), we see that the entanglement depends on effective opto-mechanics coupling strength $\zeta = D\frac{\sqrt{2\kappa_1}E}{i\Delta_s+\kappa_1+\kappa_0}$ and $\eta = R\frac{E}{\sqrt{\kappa_1}}$. If the stationary condition is fulfilled, we could say, the entanglement is enhanced by increasing ζ and η. Thus, there is a optimal coupling strength as a result of the trad-off between the energy dissipation and transferring introduced by the waveguide.

Since there are two different kinds of interactions (DC and RC) in our system, we should investigate the role of them in generating entanglement. As seen from Fig.3(a), we compares the E_N in three different conditions: (1) both DC and RC are existing, (2) only DC and (3) only RC. The results demonstrated that, the RC do not influence or generate the entanglement in the microdisk-nanostring system proposed in this paper. However, if we can increase the RC coupling strength r, as shown in Fig.3(b), the entanglement appears. This means that the RC could also lead to entanglement, which would be significant in other systems which have strong RC coupling.

In Fig. 2(b), we study the temperature dependence of E_N. Here, we choose $Q_m = 10^6$ at room temperature, which have been demonstrated in experiments [12]. It can be seen that at the optimal coupling condition, with increasing the temperature (T), the E_N decreases and the working area is also shrank. It is surprised that, the entanglement is robust against the temperature, even at room temperature (300 K), the optomechanical entanglement can be well preserved. In other words, the macroscopic oscillator can be entangled with a cavity mode field at room temperature by carefully choosing the coupling condition and frequency of pumping light. It seems quite wired that the entanglement could be preserved at such high temperature, because when the enviroment temperature largely exceeds the energy of the one quantum, i.e. $k_B T > \hbar\omega_m$, the quantum signature would be masked by the thermal noise. This counterintuitive phenomena can be understood as follow: The system's interaction with the thermal bath is governed by the mechanical dissipation rate $\gamma_m = \omega_m/Q_m$. When the interaction between the light and oscillator is much bigger than the rate of relaxation to thermal equilibrium, the quantum entanglement could be survived. If we assume $\gamma_m = 0$ ($Q_m = \infty$), the quantum system is total isolated from

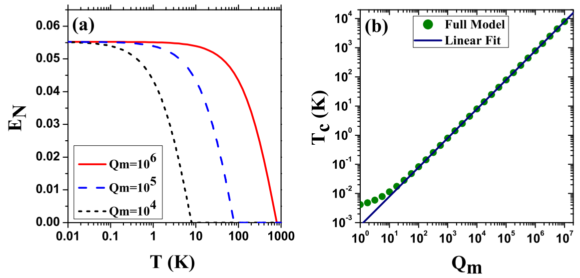

FIG. 4: (color online) (a) E_N vs the temperature T, with different Q_m. (b)The critical temperature T_C as a function of Q_m, the line is a linear fit to the full model data. Parameters are $P = 100$ mW, $\kappa_1/\kappa_0 \approx 0.3$ and $\Delta_s = 1.8\omega_m$.

the thermal bath, and the quantum behavior could be preserved at any temperature.

To study the dependence of entanglement on Q_m, in Fig.4(a), we further plot E_N against T with different Q_m, by assuming all parameters do not change with temperature. Benefitting from large $Q_m = 10^6$ of SiN nanostring in our system, the entanglement vanished at a very high critical temperature T_C. However, for lower Q_m, T_C is decreased, which corresponds to former results that entanglement only survived in cryostat [6]. It is not difficult to find in Fig.4(a) that, all curves are the same shape for different Q_m.

In order to get a better physical picture, we analysis Eq.(4) and Eq.(5) qualitatively. When Q_m is very large, $\gamma_m/\omega_m = 1/Q_m \ll 1$, and $\gamma_m/\kappa \ll 1$, we can approximately omit the γ_m in matrix M of Eq.(4). In this simplified model, from equation (5), we can obtain the dependence of solution of correlation matrix (V) on the temperature T, which is only related to I, and depends on the parameter $U(Q_m, T) = \gamma_m(2\bar{n} + 1) \approx 2\frac{1}{(\exp(\hbar\omega_m/k_B T)-1)}\frac{\omega_m}{Q_m}$. At low temperature $T \approx 0$, $U(Q_m, 0) \approx 0$, we can see from Fig.4(a) that E_N is not related to the Q_m. At high temperature, i.e. $T \gg 1$, $U(Q_m, T) \approx \frac{k_B}{\hbar}\frac{T}{Q_m}$, then E_N is a function with $\frac{T}{Q_m}$. In Fig. 4(b), the dependence of T_C of entanglement on the Q_m shows great agreement with the relationship $\frac{T}{Q_m} = const$ when $Q_m > 100$.

Conclusion.- A potential experimental system has been proposed to generate stationary room-temperature optomechanical entanglement. The system consists of a high-Q nanostring oscillator and a microdisk cavity, fabricated by SiN, which can be integrated on single chip. The role of both dispersive and reactive coupling in generating optomechanical entanglement is studied. We find that in the system proposed in this paper, the room-temperature entanglement is effectively obtained through the dispersive couplings. In particular, we find, in the high Temperature (T) and high mechanical qualify factor (Q_m) limit, the logarithmic entanglement depends only on the ratio T/Q_m. This means that improvements in material quality, and optimization in the structure design may enable both Q_o and Q_m greater, and lead to more efficient generation of stationary entanglement. This system can be applied as fundamental elements on photon chips[18–20]. With a rapid development in material science and fabrication technology, the scheme proposed here is expected to be realized in the near future, and to explore the high-temperature entanglement of macroscopic objects [21].

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[1] The Physics of Quantum Information, edited by D. Bouwmeester, A Ekert, and A. Zeilinger (Springer, Berlin, 2000).
[2] A. Einstein et al., Phys. Rev. **47**, 777 (1935).
[3] P. G. Kwiat et al., Phys. Rev. Lett. **75**, 4337 (1995); R. Reichle et al., Nature **443**, 838 (2006); E. Togan et al., Nature **466** , 730 (2010); V. Vedral, Nature **453**, 1004 (2008).
[4] K. C. Schwab et al., Phys. Today **58**(7), 36 (2005).
[5] T. J. Kippenberg and K. J. Vahala, Opt. Express **15**, 17172 (2007); M. Aspelmeyer et al., J. Opt. Soc. Am. B **27**, A189 (2010); D. V. Thourhout and J. Roels, Nat. Photon. **4**, 211 (2010).
[6] D. Vitali et al., Phys. Rev. Lett. **98**, 030405 (2007).
[7] M. J. Hartmann and M. B. Plenio, Phys. Rev. Lett. **101**, 200503 (2008).
[8] D. Vitali et al., Phys. Rev. A **76**, 042336 (2007).
[9] M. Li et al., Phys. Rev. Lett. **103**, 223901 (2009).
[10] G. Anetsberger et al., Nat. Phys. **5**, 909 (2009); G. Anetsberger, et al., Phys. Rev. A **82**, 061804 (2010).
[11] P. Barclay et al., Appl. Phys. Lett. **89**, 131108 (2006); E. S. Hosseini et al., Opt. Express **17**, 14543 (2009).
[12] S. S. Verbridge et al., Appl. Phys. Lett. **92**, 013112 (2008); K. Y. Fong et al., Appl. Phys. Lett. **97**, 073112 (2010); J. B. Hertzberg et al., Nature Phys. **6**, 213 (2010).
[13] M. L. Gorodetsky and V. S. Ilchenko, J. Opt. Soc. Am. B **16**, 147 (1999).
[14] D. F. Walls and G. J. Milburn, Quantum Optics, (Springer, Berlin, 1994).
[15] V. Giovannetti, D. Vitali, Phys. Rev. A **63**, 023812 (2001).
[16] E. X. DeJesus, C. Kaufman, Phys. Rev. A **35**, 5288 (1987).
[17] G. Vidal et al., Phys. Rev. A **65**, 032314 (2002).
[18] L. Tian and H. Wang, Phys. Rev. A **82**, 053806 (2010).
[19] K. Stannigel et al., Phys. Rev. Lett. **105**, 220501 (2010).
[20] J. O'Brien et al., Nature Photon. **3**, 687 (2009).
[21] F. Galve et al. Phys. Rev. Lett. **105**, 180501 (2010).